Tutorial 2 (ComplexNum+TrigIdentities)

(You should practise writing proper steps.)

1. Let z_1 and z_2 be two complex numbers given as $z_1 = 2-3i$ and $z_2 = 1+2i$.
   Compute the following.
   (a) $z_1\bar{z}_1$
   (b) $z_1 z_2$
   (c) $(z_1 + 3z_2)^2$
   (d) $[z_1 + (1+z_2)]^2$

2. Express the following in the form $a+bi$, where a and b are real numbers.
   (a) $\dfrac{1+4i}{5-12i}$
   (b) $(2+i)^3$
   (c) $3\sqrt{-50}+\sqrt{-72}$
   (d) $\dfrac{1}{5-3i}-\dfrac{1}{5+3i}$

3. (a) What is Euler's formula?

   (b) Use Euler's formula to derive the identities
   (i) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
   (ii) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

4. Prove the identities
   (i) $\sin 2\theta = 2\sin\theta\cos\theta$
   (ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
   in two ways as described below.

   (a) By setting $\alpha = \theta$ and $\beta = \theta$ in the identities
   $$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
   $$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

   (b) By using Euler's formula and the identity $e^{i(2\theta)} = \left(e^{i\theta}\right)^2 = e^{i\theta}\cdot e^{i\theta}$

5. Derive the identity that expressses $\sin 3\theta$ in terms of $\sin\theta$ and $\cos\theta$ by using Euler's formula and the identity $e^{i(3\theta)} = \left(e^{i\theta}\right)^3$.

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7. Express $\sin 4x \sin 5x$ in a form involving the difference of two cosines.

8. Express $\cos 5x \cos 2x$ in a form involving the sum of two cosines.

9. Express $\sin 5x \cos 2x$ in a form involving the sum of two sines.

10. Express $\cos 5x \sin 3x$ in a form involving the difference of two sines.

11. Derive the subtraction formulas

(i) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

(ii) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

in two ways as described below.

(a) By replacing β with $-\beta$ in the following two identities you have derived earlier

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

(b) By using Euler's formula and the identity $e^{i\alpha}\cdot e^{-i\beta} = e^{i(\alpha-\beta)}$

12. Obtain the addition formula and subtraction formula for tangent.

Use the addition and subtraction formulas for sine and cosine to express $\tan(\alpha+\beta)$ and $\tan(\alpha-\beta)$ in terms of $\tan\alpha$ and $\tan\beta$.

[Note that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.]

13. Use the special angles $\alpha = \dfrac{4\pi}{3}$ and $\beta = \dfrac{\pi}{3}$ to verify that all the addition and subtraction formulas for sine, cosine and tangent hold.

[*You may need help in understanding what this question asks*.]

(nby, Nov 2015)